# Testing gravity

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Non-Linear Structure in the Modified Universe
Lorentz Center Leiden
July 2014

# Testing gravity with relativistic effects

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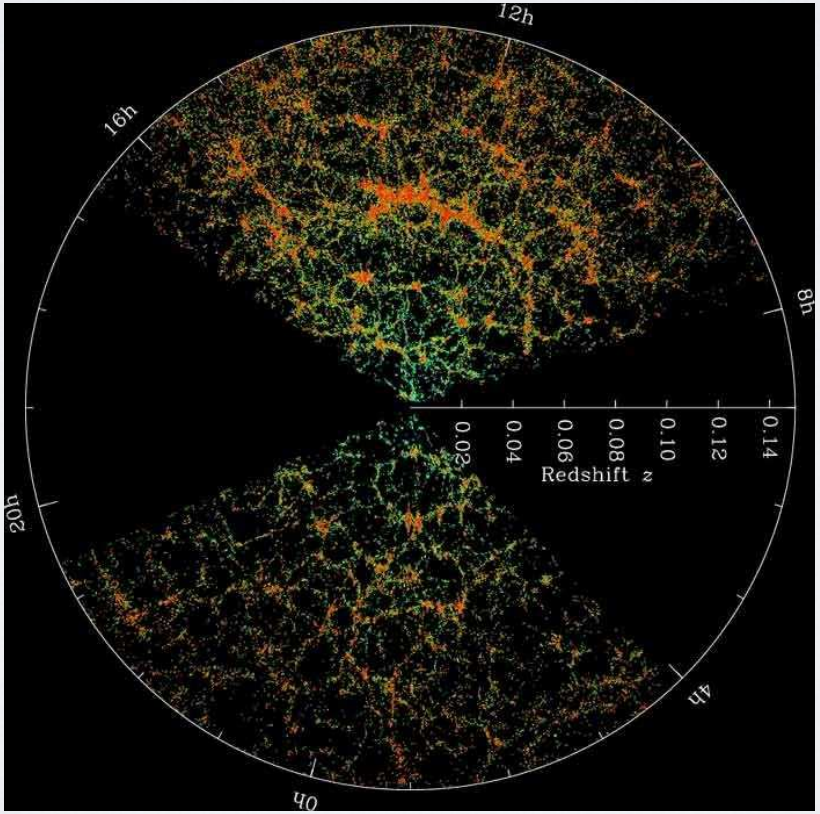
# Outline

- What are **relativistic distortions**?
  - → Effect on: the galaxy number counts $\Delta$
    the convergence $\kappa$ (or magnification)
- How can we **measure** relativistic effects?
  - → We can isolate these effects by looking at **anti-symmetries** in the correlation function.
- How can we use them to **test gravity**?

# Galaxy survey

- We want to measure **fluctuations** in the distribution of galaxies.
- We pixelise the map.
- We count the number of **galaxies** per **pixel**: $\Delta = \dfrac{N - \bar{N}}{\bar{N}}$
- Question: what are the effects contributing to $\Delta$ ?

Credit: M. Blanton, SDSS



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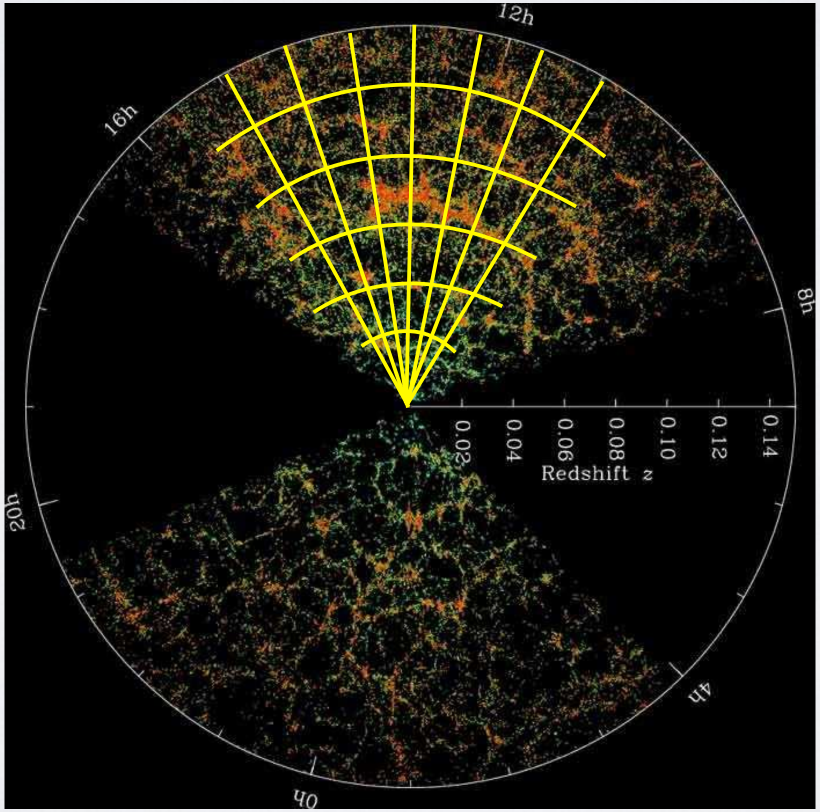

# Galaxy number counts

We observe $\Delta(z,\mathbf{n}) = \dfrac{N(z,\mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}$

$N(z,\mathbf{n}) = \rho(z,\mathbf{n})V(z,\mathbf{n})$ and $\bar{N} = \bar{\rho}(z)\bar{V}(z)$

$$\Delta = \frac{\rho(z,\mathbf{n}) \cdot V(z,\mathbf{n}) - \bar{\rho}(z) \cdot \bar{V}(z)}{\bar{\rho}(z) \cdot \bar{V}(z)}$$

$$\boxed{\Delta(z,\mathbf{n}) = b \cdot \delta(z,\mathbf{n}) + \frac{\delta V(z,\mathbf{n})}{V} - 3\frac{\delta z}{1+z}}$$

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$\rho = \bar{\rho} + \delta\rho$ $\quad V = \bar{V} + \delta V$ $\quad z = \bar{z} + \delta z$

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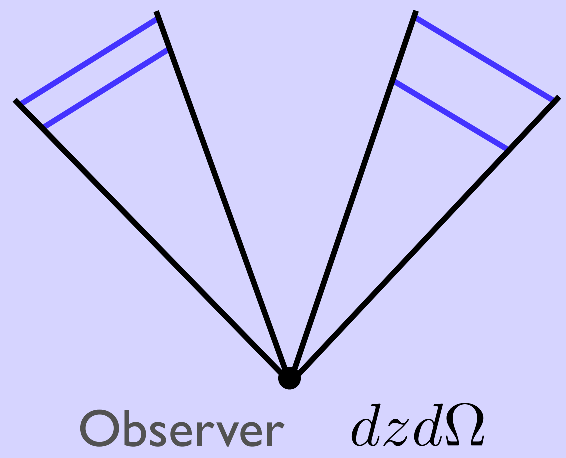
same redshift bin
different physical volume

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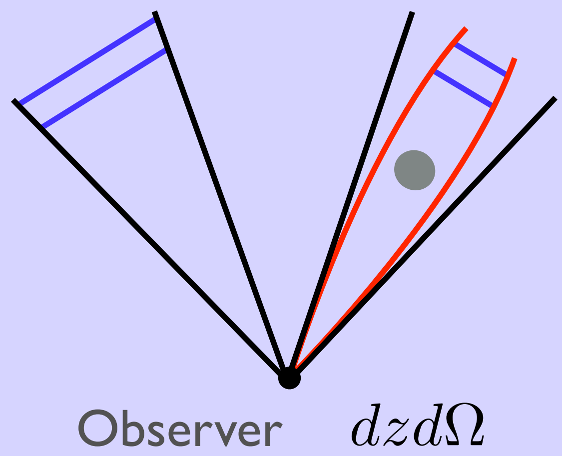
$$\Delta = \frac{\rho(z,\mathbf{n})\cdot V(z,\mathbf{n}) - \bar{\rho}(z}{\bar{\rho}(z)\cdot\bar{V}(z)}$$

same solid angle
different physical volume

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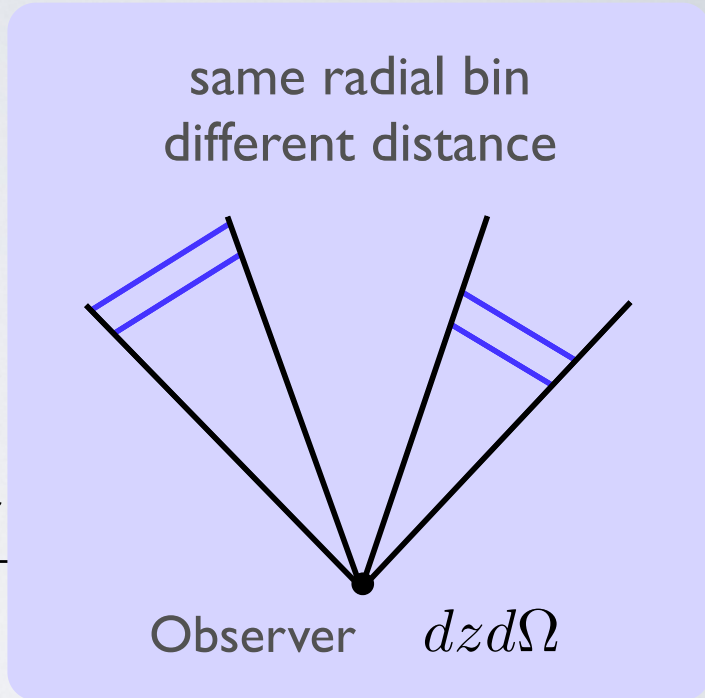


$\rho = \bar{\rho} + \delta\rho$ $\qquad$ $V = \bar{V} + \delta V$

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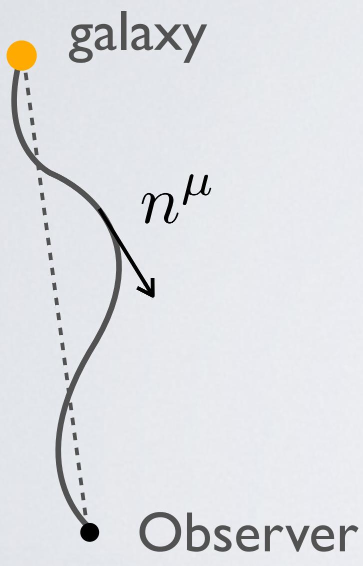
$$\boxed{\Delta(z,\mathbf{n}) = b \cdot \delta(z,\mathbf{n}) + \frac{\delta V(z,\mathbf{n})}{V} - 3\frac{\delta z}{1+z}}$$

# Fluctuations

Perturbed Friedmann universe:

$$ds^2 = -a^2\Big[\big(1+2\Psi\big)d\eta^2 + \big(1-2\Phi\big)\delta_{ij}dx^i dx^j\Big]$$




We follow the **propagation** of **photons** from the galaxies to the observer and calculate:

- Changes in **energy**
- Changes in **direction**

# Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

density

redshift space distortion

lensing

gravitational redshift

Doppler

$$\Delta(z,\mathbf{n}) = b\cdot D - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$
$$- \int_0^r dr' \frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$
$$+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi$$
$$\left.\begin{aligned} &+ \Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi) \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right] \end{aligned}\right] \rightarrow \text{potential}$$

# Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

standard expression

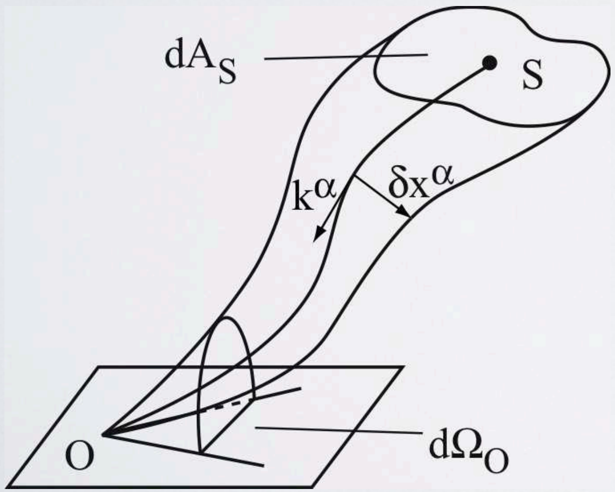
lensing: important at high z

relativistic contributions:
important at large scale

$$\Delta(z,\mathbf{n}) = \boxed{b\cdot D - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})}$$
$$\boxed{-\int_0^r dr' \frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)}$$
$$\boxed{\begin{aligned}&+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi\\&+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi)\\&+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]\end{aligned}}$$

# Convergence

- Galaxy surveys observe also the shape and the luminosity of galaxies → measure of the **convergence**.
- The convergence $\kappa$ measures distortions in the **size**.
- The shear $\gamma$ measures distortions in the **shape**.
- **Relativistic** distortions affect the convergence at **linear** order.




We solve Sachs equation

$$\frac{D^2\delta x^\alpha(\lambda)}{D\lambda^2} = R^\alpha{}_{\beta\mu\nu}k^\beta k^\mu \delta x^\nu$$

# Convergence

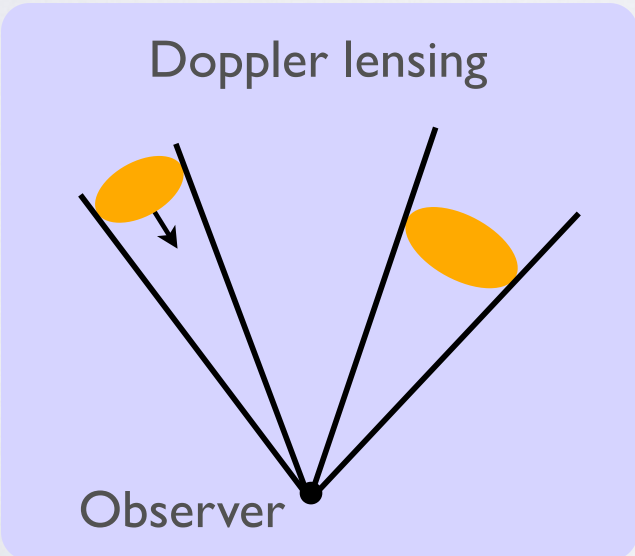
CB (2008)
Bolejko et al (2013)
Bacon et al (2014)

Gravitational lensing

Doppler lensing

$$\kappa = \frac{1}{2r}\int_0^r dr' \frac{r-r'}{r'}\Delta_\Omega(\Phi+\Psi) + \left(\frac{1}{r\mathcal{H}}-1\right)\mathbf{V}\cdot\mathbf{n}$$
$$-\frac{1}{r}\int_0^r dr'(\Phi+\Psi) + \left(1-\frac{1}{r\mathcal{H}}\right)\int_0^r dr'(\dot{\Phi}+\dot{\Psi})$$
$$+\left(1-\frac{1}{r\mathcal{H}}\right)\Psi + \Phi \rightarrow \text{Sachs Wolfe}$$

Integrated terms




The moving galaxy is further away → it looks smaller, i.e. demagnified

# Observations

- Due to relativistic effects, $\Delta$ and $\kappa$ contain additional **information**.

$$\delta, V, \Phi, \Psi \qquad\qquad (\Phi + \Psi), V, \Phi, \Psi$$

- This can help **testing gravity** by probing the **relation** between density, velocity and gravitational potentials.

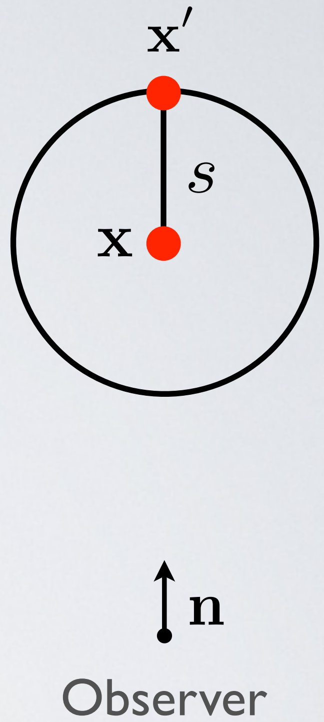
- Two **difficulties**:
  - The relativistic effects are small: we need to go to large scales.
  - We always measure the sum of all the effects.

- We need a way of **isolating** relativistic effects

  → look for **anti-symmetries** in the correlation function.

# Density

The **density** contribution $\Delta = b \cdot \delta$, generates an **isotropic** correlation function.




$\xi(s) = \langle \Delta(\mathbf{x})\Delta(\mathbf{x}') \rangle$ depends only on the **separation** $s = |\mathbf{x} - \mathbf{x}'|$

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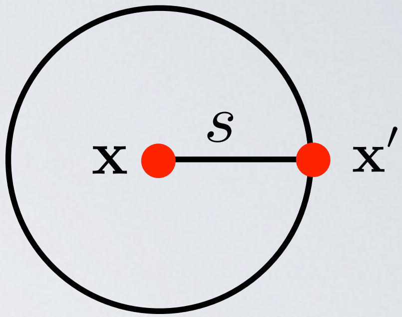


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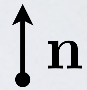
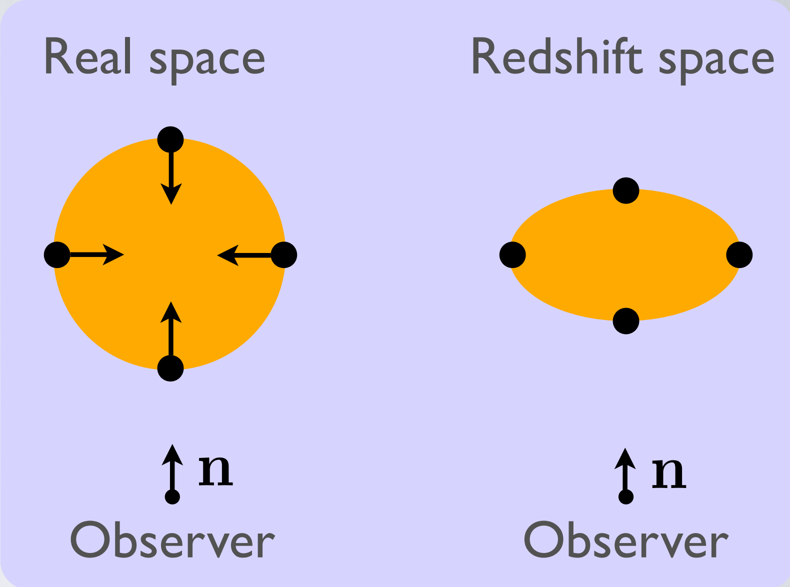


# Redshift distortions

Redshift distortions **break** the **isotropy** of the correlation function.

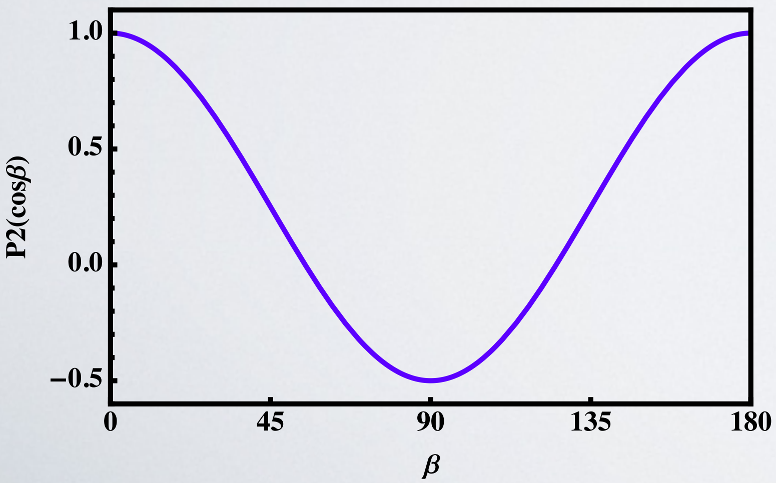
$$\Delta = b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})$$

Hamilton (1992)



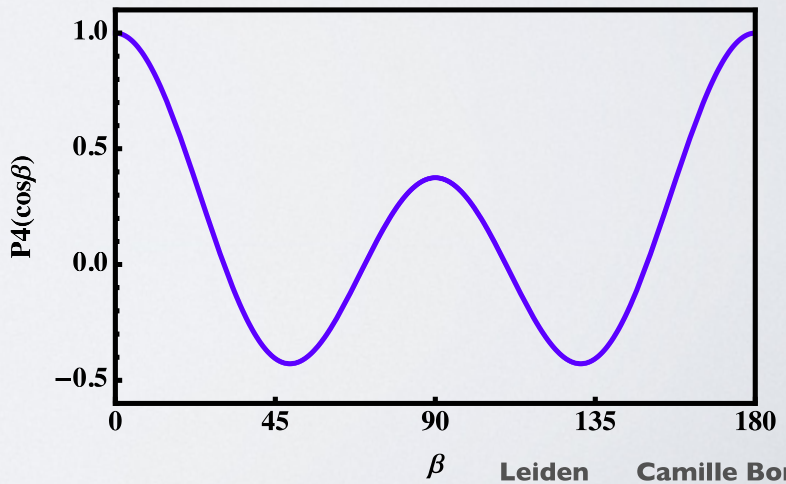

## Quadrupole

$$P_2(\cos\beta) = \frac{3}{2}\cos^2\beta - \frac{1}{2}$$




## Hexadecapole

$$P_4(\cos\beta) = \frac{1}{8}\left[35\cos^4\beta - 30\cos^2\beta + 3\right]$$



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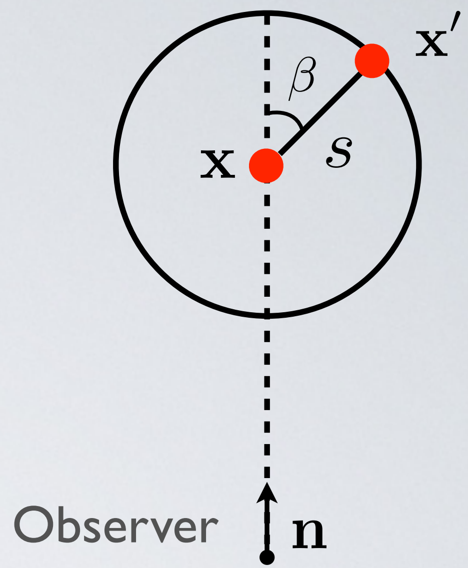


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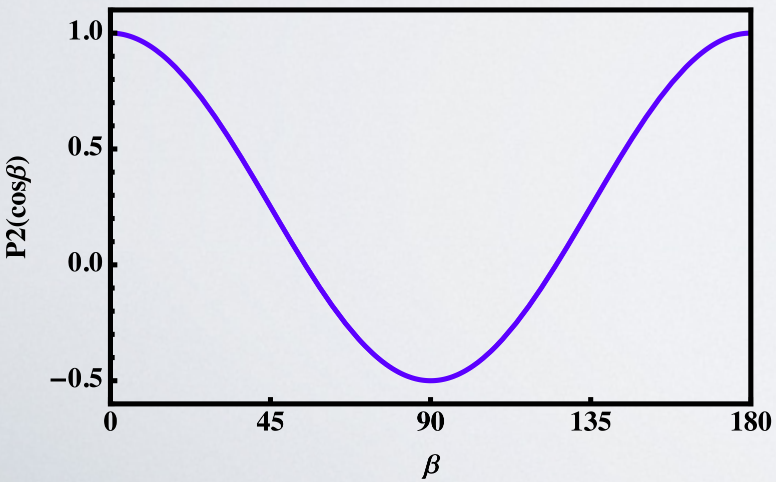


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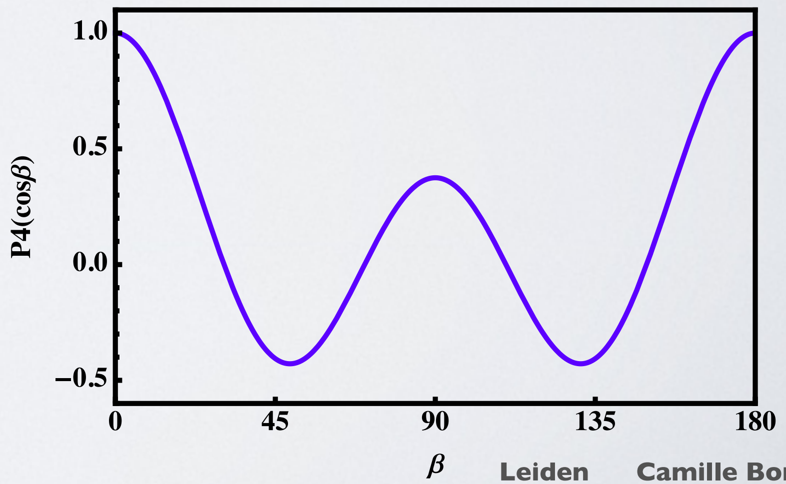
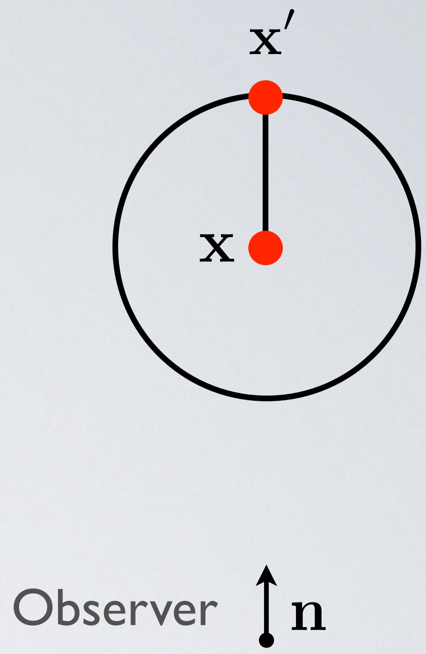


# Relativistic effects




The relativistic effects break the **symmetry** of the correlation function.

The correlation function differs for galaxies **behind** or in **front** of the central one.

This differs from the breaking of **isotropy**, due to redshift distortions, which is symmetric.

To measure the asymmetry, we need **two populations** of galaxies: faint and bright.

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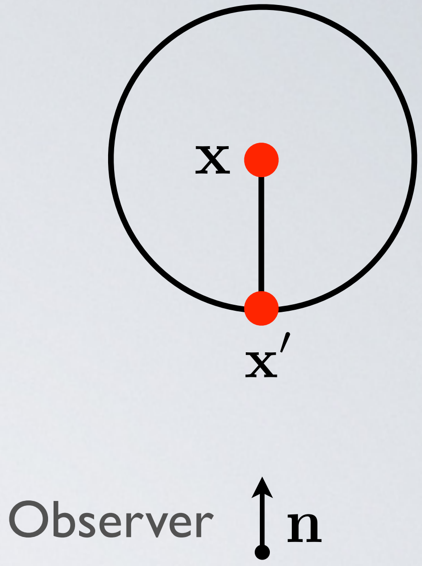


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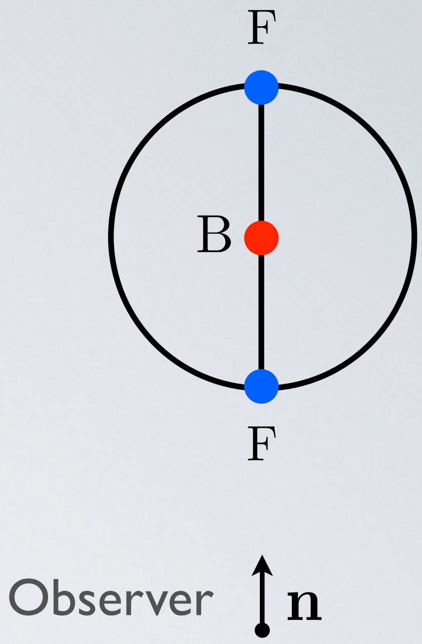


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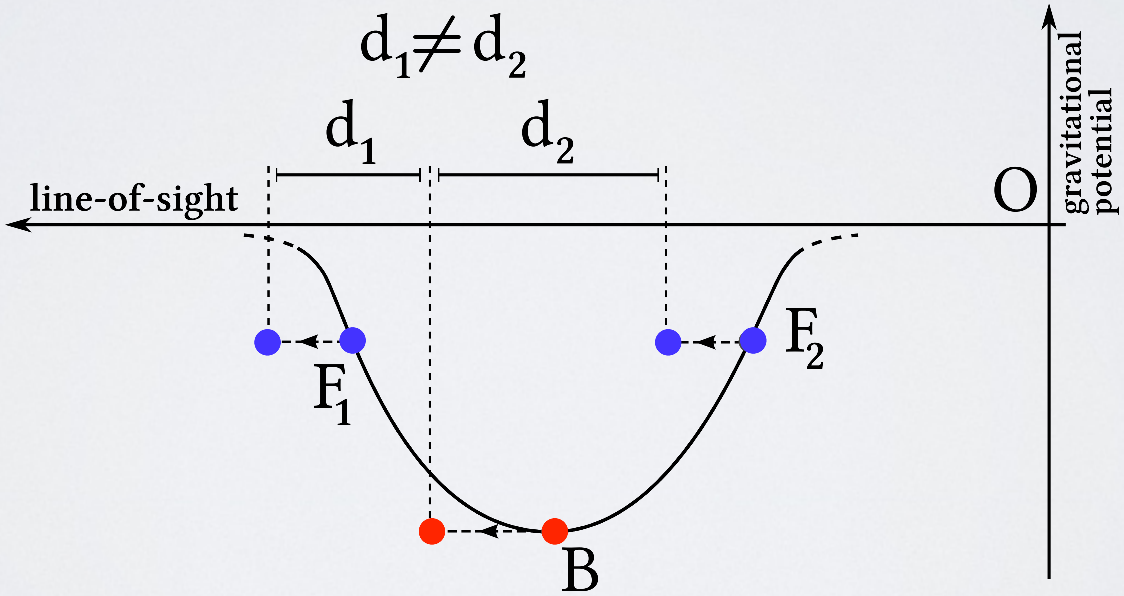
To measure the asymmetry, we need **two populations** of galaxies: faint and bright.

# Cross-correlation

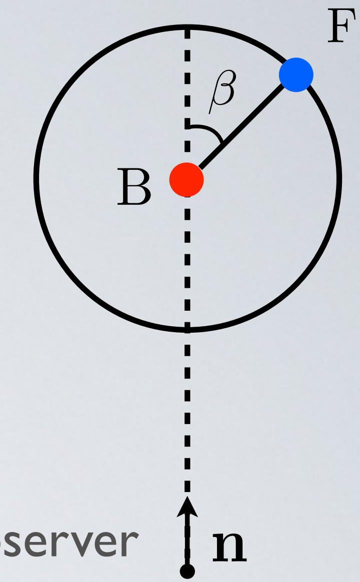
The following terms **break** the **symmetry**:

$$\Delta_{\rm rel} = \frac{1}{\mathcal{H}}\partial_r\Psi + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}$$



# Dipole in the correlation function

CB, Hui and Gaztanaga (2013)




$$\xi(s,\beta) = D_1^2\, f\, \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_{\mathrm{B}} - b_{\mathrm{F}})\nu_1(s)\cdot\cos(\beta)$$

$$\nu_1(s) = \frac{A}{2\pi^2}\int\frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k)T_\Psi(k)\, j_1(k\cdot s)$$

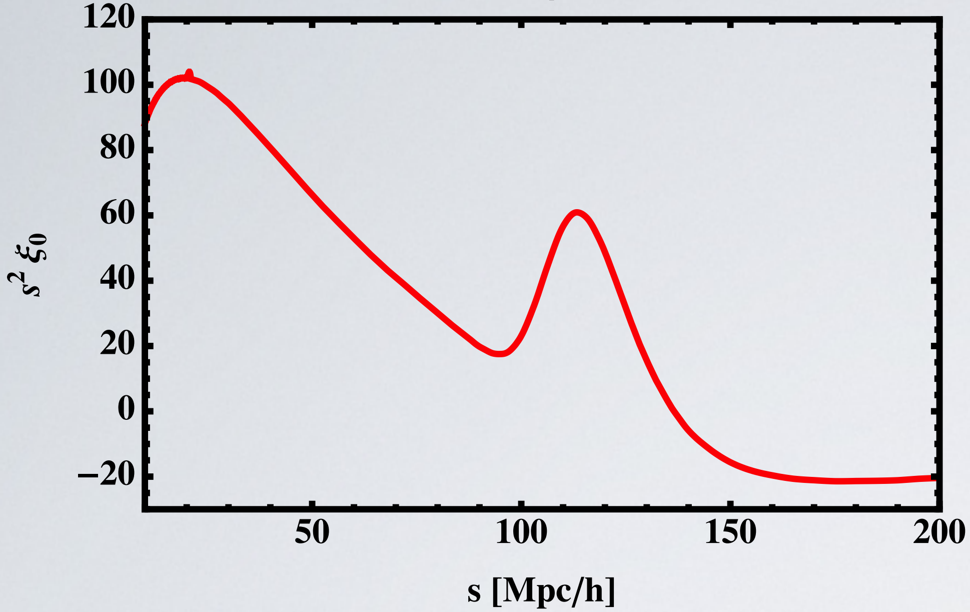
By fitting for a **dipole** in the correlation function, we can measure **relativistic effects**, and separate them from the density and redshift space distortions.

$$\boxed{\xi_1(s) = \frac{3}{2}\int_{-1}^{1} d\mu\, \xi(s,\mu)\cdot\mu \qquad \mu = \cos\beta}$$

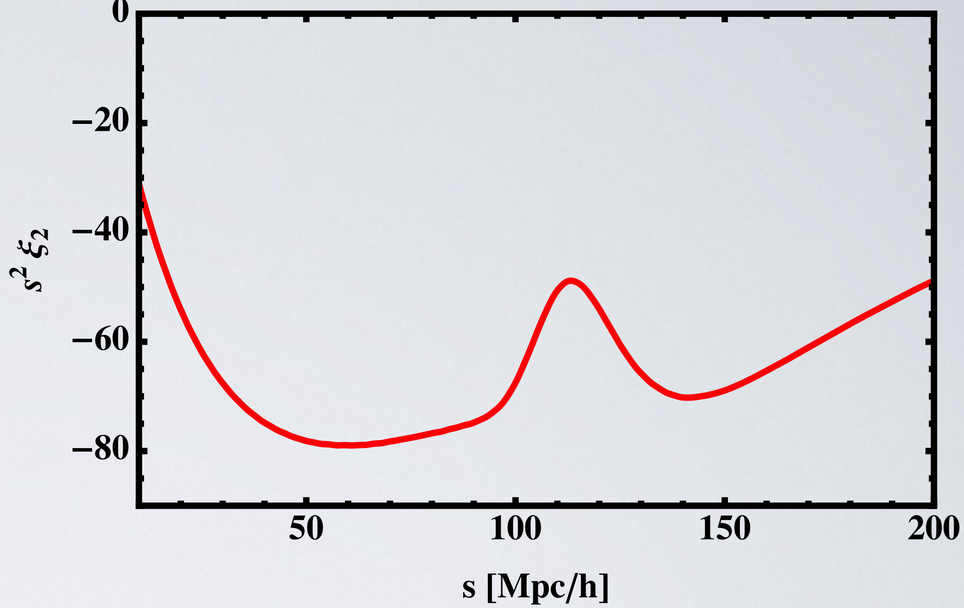
$z = 0.25$

# Multipoles

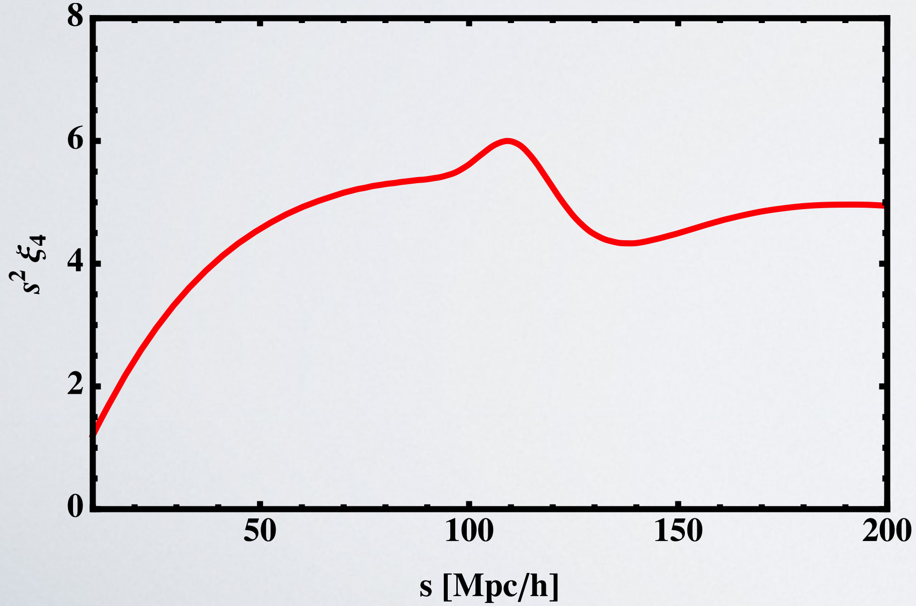
Monopole



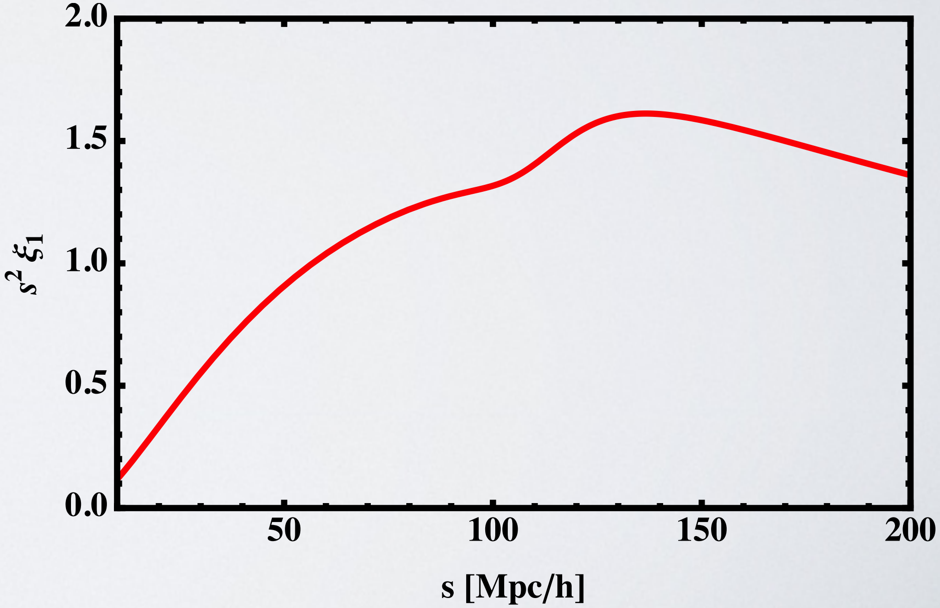

Quadrupole




Hexadecapole



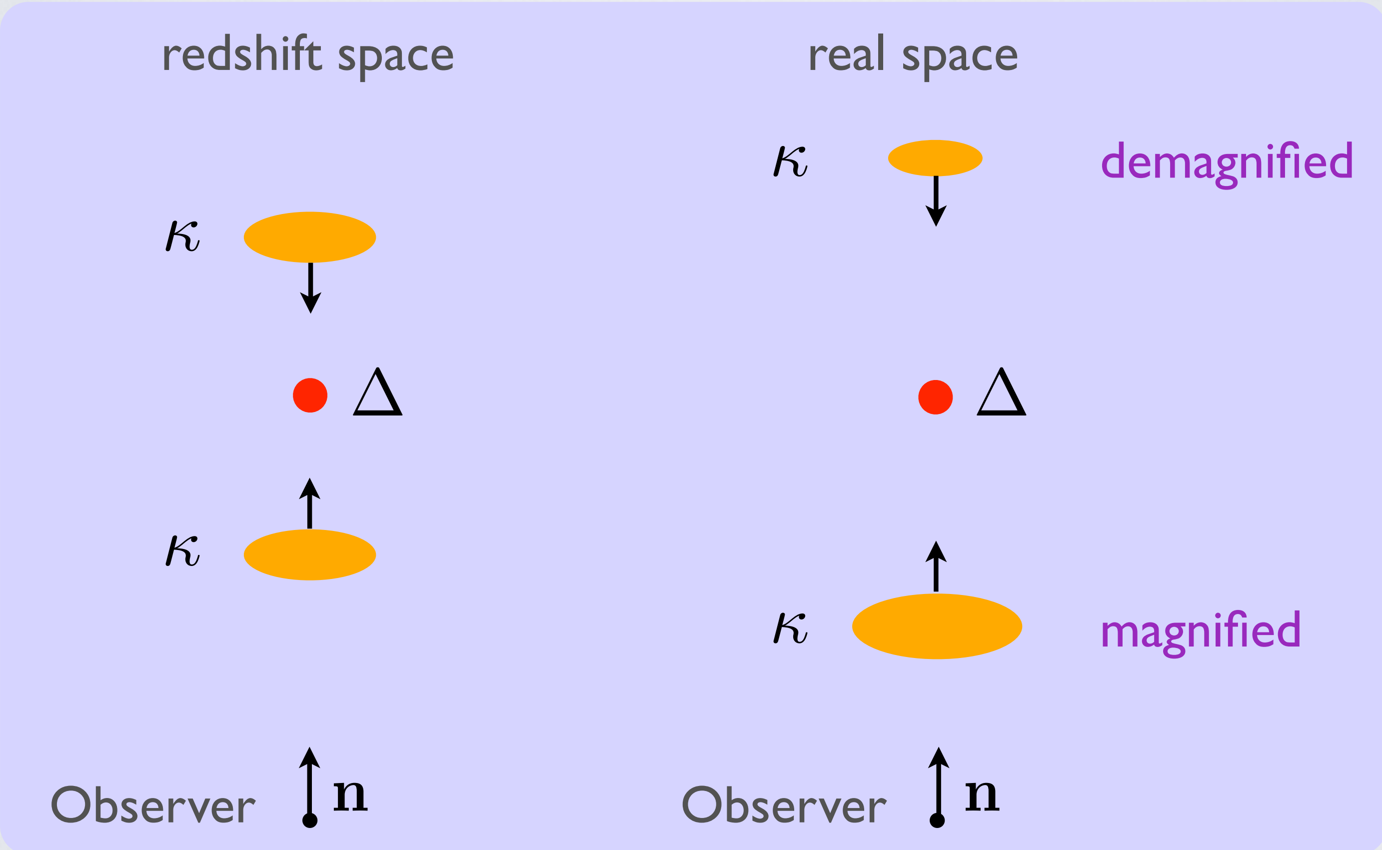

Dipole



# Convergence

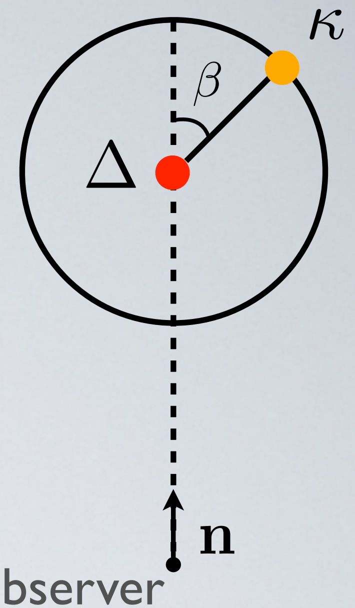
$$\kappa_{\rm g} = \frac{1}{2r}\int_0^r dr' \frac{r-r'}{r'}\Delta_\Omega(\Phi+\Psi) \qquad \kappa_{\rm v} = \left(\frac{1}{r\mathcal{H}} - 1\right)\mathbf{V}\cdot\mathbf{n}$$

We can isolate the Doppler lensing by looking for **anti-symmetries** in $\langle\Delta\kappa\rangle$



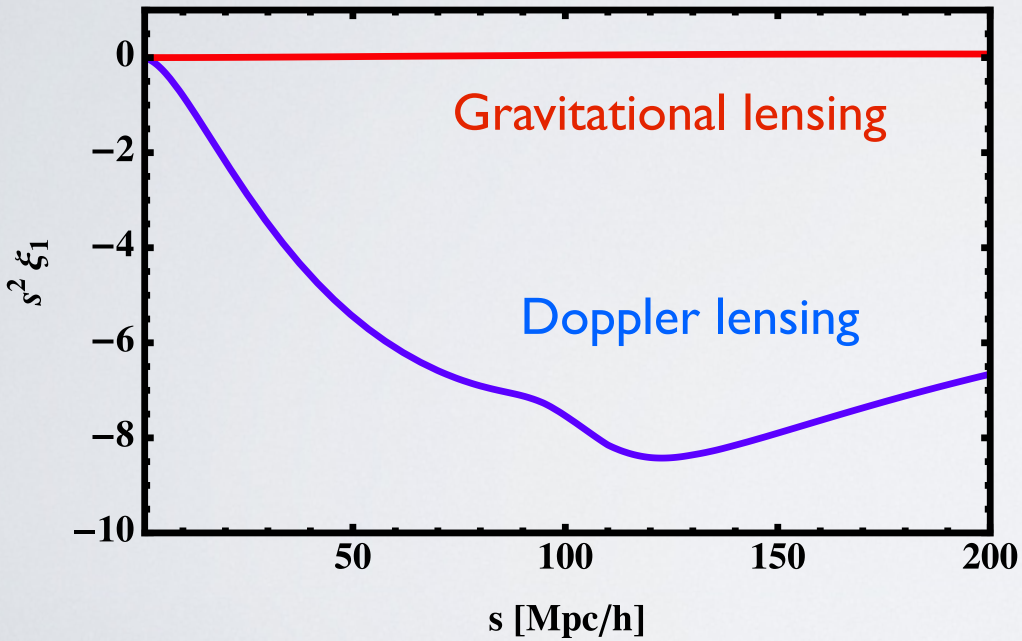
# Dipole

CB, Bacon, Clarkson, Andrianomena and Maartens (in preparation)

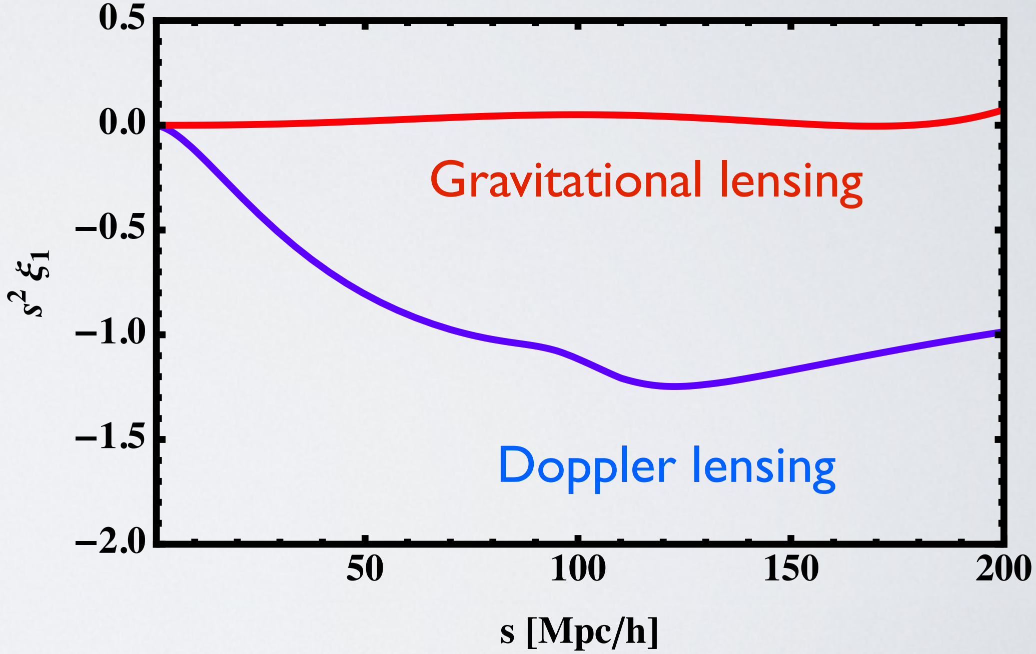



$$\xi(s,\beta)=\frac{2A}{9\pi^2\Omega_m^2}D_1^2\frac{\mathcal{H}}{\mathcal{H}_0}f\left(1-\frac{1}{\mathcal{H}r}\right)\left(b+\frac{3f}{5}\right)\nu_1(s)\cos(\beta)$$

$z=0.1$




$z=0.5$




The dipole due to gravitational lensing is completely subdominant.

# Testing Euler equation

- The monopole and quadrupole in $\Delta$ allow to measure $V$

- The dipole allows to measure:

$$\Delta_{\rm rel} = \frac{1}{\mathcal{H}}\partial_r\Psi + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}$$

- If Euler equation is valid: $\dot{\mathbf{V}}\cdot\mathbf{n} + \mathcal{H}\mathbf{V}\cdot\mathbf{n} + \partial_r\Psi = 0$

$$\Delta_{\rm rel} = -\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}$$

→ With the dipole, we can **test** Euler equation.

# Measuring the anisotropic stress

- The dipole in the convergence is sensitive to:

$$\kappa_{\rm v} = \left(\frac{1}{r\mathcal{H}} - 1\right)\mathbf{V}\cdot\mathbf{n}$$

- The standard part $\kappa_{\rm g} = \frac{1}{2r}\int_0^r dr'\frac{r-r'}{r'}\Delta_\Omega(\Phi+\Psi)$

  can be measure through $\langle\kappa\kappa\rangle$ and $\langle\gamma\gamma\rangle$

- Assuming Euler equation, we can test the relation between the two metric potentials $\Phi$ and $\Psi$.

# Conclusion

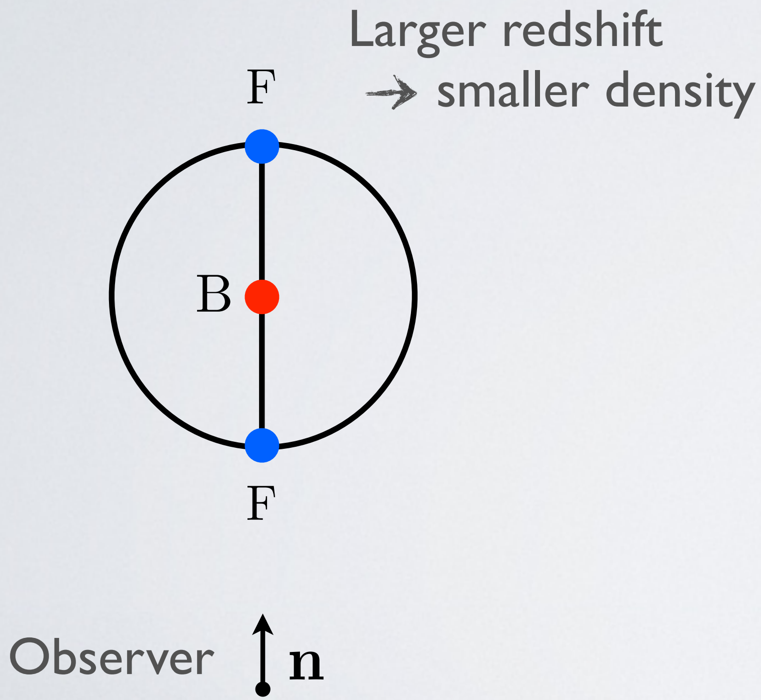
- Our **observables** are affected by relativistic effects.

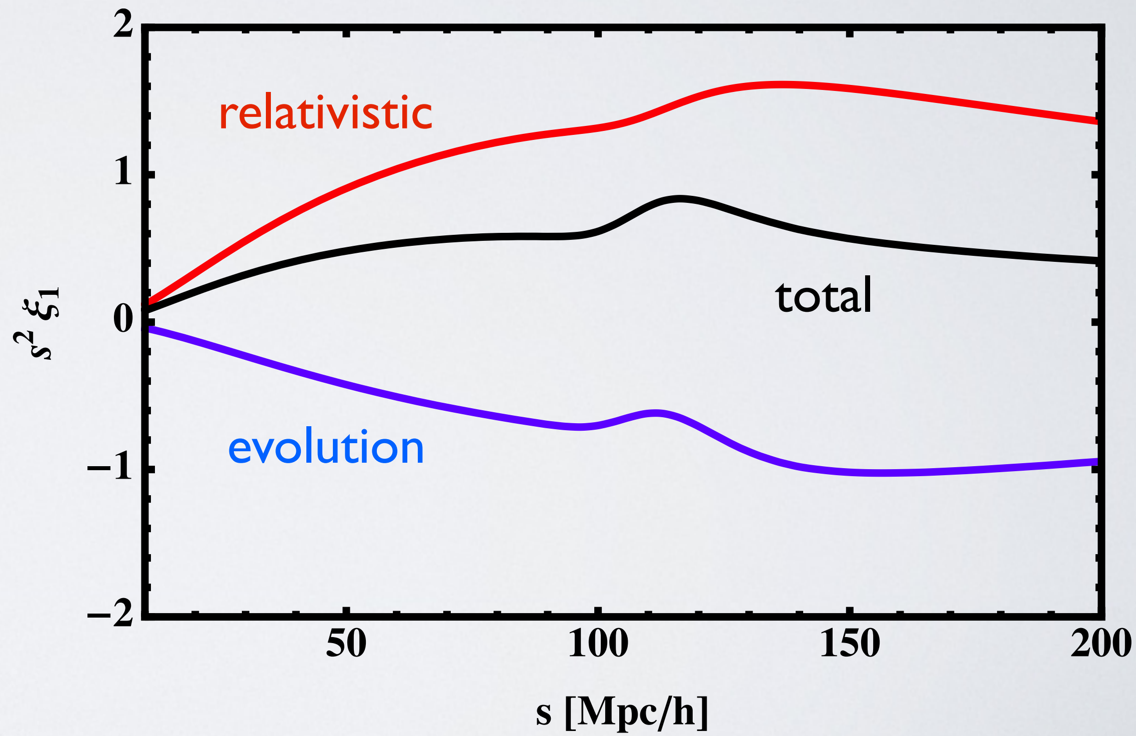
- These effects have a different **signature** in the **correlation** function: they induce anti-symmetries.

- By measuring these anti-symmetries we can isolate the relativistic effects and use them to **test** the **relations** between the density, velocity and gravitational potential.

# Contamination

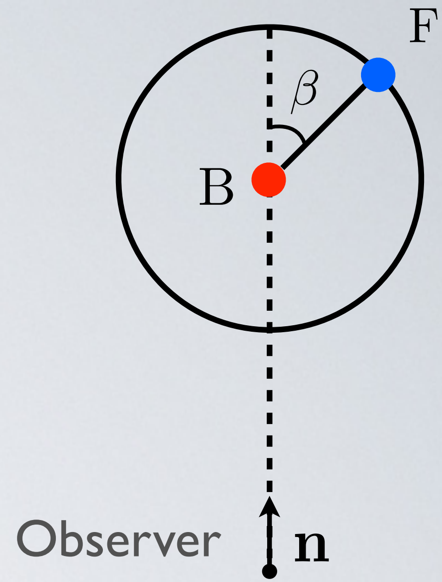
The density and velocity **evolve** with time: the density of the faint galaxies in front of the bright is larger than the density behind. This also induces a **dipole** in the correlation function.



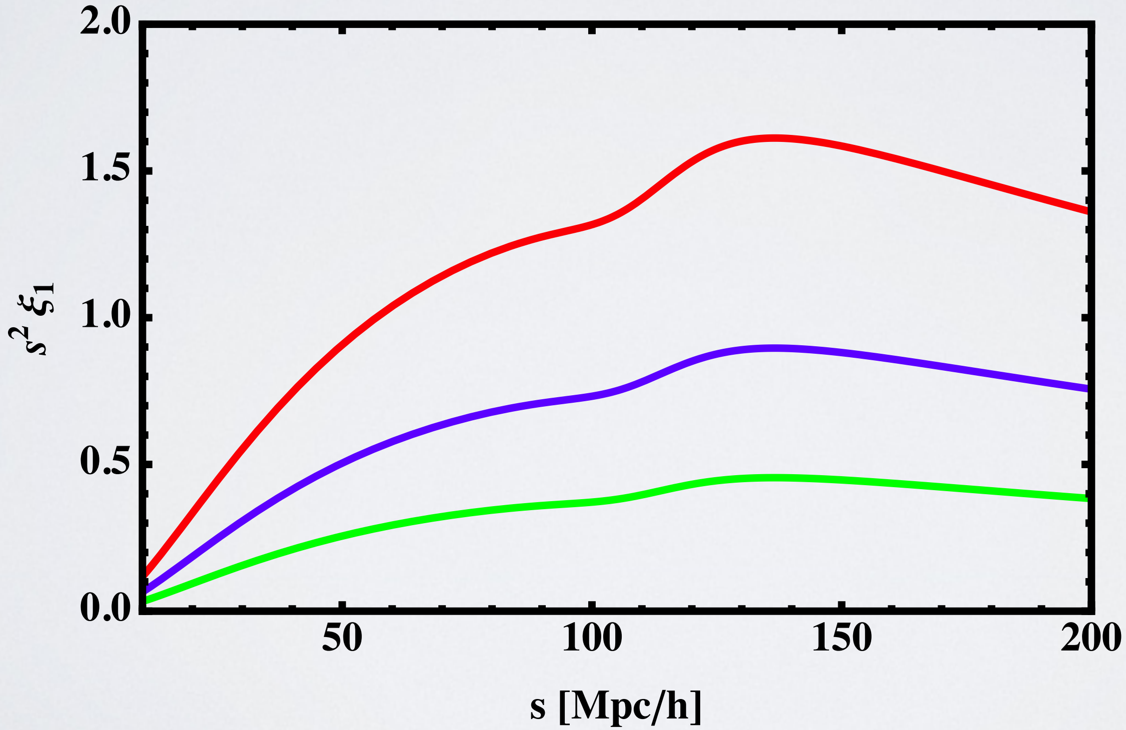



# Dipole in the correlation function

$$\xi(s,\beta) = D_1^2\, f\, \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_{\rm B} - b_{\rm F})\nu_1(s)\cdot \cos(\beta)$$




$$\nu_1(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k) T_\Psi(k)\, j_1(k\cdot s)$$




$z = 0.25$

$z = 0.5$

$z = 1$

$b_{\rm B} - b_{\rm F} \simeq 0.5$

CB, Hui and Gaztanaga (2013)

# Multipoles

- Monopole $$\xi_0 = D_1^2 b^2 \mu_0(s)$$
- Quadrupole $$\xi_2 = -D_1^2 \left(\frac{4fb}{3} + \frac{4f^2}{7}\right) \mu_2(s) P_2(\cos\beta)$$
- Hexadecapole $$\xi_4 = D_1^2 \frac{8f^2}{35} \mu_4(s) P_4(\cos\beta)$$

$$\mu_\ell(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left(\frac{k}{H_0}\right)^{n_s-1} T_\delta^2(k)\, j_\ell(k\cdot s)$$